Theoretical Formulation and Finite Elemental Analysis of the Conformal Cylindrical Contact

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***Abstract* - It is necessary to examine the contact stresses developed between the contacting bodies. The contact stress depends on the material property of the contact bodies, clearance between them and load acting on it. The combination of contacting materials and contact conditions are critical. The effect of clearance and load on the contact conditions of cylindrical conformal contact is examined for same and varies values of Young's modulus of the two contacting bodies. Finite element analysis is used to study cylindrical conformal contacts with the models and is validated by comparison with known analytical solutions. These results can be applied to any problem of conformal contact in nature such as actuators, hip-joint, piston barrel, cylinder block and the piston ball, slipper seat of a water-lubricated axial piston pump and for solid structural bearing with shaft.**

***Index Terms* - Contact Stress, Conformal Cylindrical Contact, Contact Angle, Contact Clearance, Half Space.**

I. Introduction

The determination of pressure at a contact in a machine part is important because contact stresses frequently lead to failure by seizure, wear or fatigue. Due to a small radial clearance and a heavy load application in certain mechanical joints a conformal (Non-Hertizian) type of contact exists. Under the application of the load the size of the contact area grows rapidly and may become comparable with the significant dimensions of the contacting bodies. A pin in a hole with a small clearance is an example.When the area of contact occupies an appreciable fraction of the circumference of the hole neither the pin nor the hole can be regarded as an elastic half-space so that the Hertz treatment is invalid. But still it can be considered as half-space for the purpose of calculating elastic deformations and stresses.

Many problems of contact mechanics are solved using the complex integration are given by N.I.Muskhelishvili in the year 1963[1]. The finite element analysis and approximate model for cylindrical joints with clearances are given by Cai-Shan Liu, Ke Zhang and Rei Yang in 2006 [2] where By introducing some appropriate assumptions and analyzing the FEM numerical results, such as the contact area, the pressure distribution, and the maximum sustainable load, an approximate model for the contact problem of cylindrical joints with clearances is developed through modeling the pin as a rigid wedge and the elastic plate as a simple Winkler elastic foundation [3].

There are numerous methods to solve contact problems. Some of the different methods of contact stress evolution are analytical closed form solution, numerical solutions and experimental methods. It is highly desirable to obtain complete closed-form analytical solutions to problems in contact mechanics. An analytic result means, that the full internal stress-displacement field for the given situation is expressed in a closed form in terms of known elementary (polynomial, exponential, etc.) and special (elliptical, Bessel, etc.) functions [4]. The numerical techniques are powerful tools to analyze the contact problem because of their flexibility and ability to model all complications involved with the analysis of such problems.

Recently, several contact algorithms have been proposed and incorporated into commercially available Finite Element software. In the contact zones existing in various mechanisms, stresses due to quasi-static loads can lead to friction and to high stresses thereby inducing fatigue and erosion of surfaces. In order to design these elements it is necessary to predict contact actions and stresses. Exact solutions of contact problems are available for ideal conditions [5,6,7].

The combined effects on the stress concentrations of the shaft radius, the interference, and Young's modulus of shaft and hub with the aid of finite elements; various design plots are compiled that report the elastic stress concentrations, contact pressure within the hub versus the normalizing parameter like contact angle and clearance between the shaft and hub. In most of the actual situations analytical solutions don't exists and approximate solutions have to be obtained, using either Finite Element method formulation or a variation formulation.

II. THEORETICAL FORMULATION

Steumann found the distribution of pressure $P_n(x)$ for profiles having the form $A_n x^{2n}$. For the two dimensional half-space problems in which the displacements are specified over

DOI: 01.IJPIE.02.01.508

the interval $-a \le x \le a$, the singular integral equation is obtained as

$$\int \frac{F(s)}{x-s}\,ds = g(x) = A_n x^{2n} \qquad (1)$$

Here F(s) is the component of traction. The resulting equation is

$$P_n(x) = \frac{P_n}{\pi\sqrt{a^2-x^2}} - \frac{E^* n A_n a^{2n}}{\sqrt{a^2-x^2}}\left\{\left(\frac{x}{a}\right)^{2n} - \frac{1}{2}\left(\frac{x}{a}\right)^{2n-2} - \frac{1.3....(2n-3)}{2.4....2n}\right\} \qquad (2)$$

To avoid infinite pressure at $x = \pm a$

$$P_n = E^* n A_n a^{2n} \frac{1.3....(2n-3)}{2.4....2n} \qquad (3)$$

Therefore the pressure distribution becomes

$$P_n(x) = E^* n A_n a^{2n-2}\left\{\left(\frac{x}{a}\right)^{2n-2} + \frac{1}{2}\left(\frac{x}{a}\right)^{2n-4} + ... + \frac{1.3....(2n-3)}{2.4....(2n-2)}\right\}\sqrt{a^2-x^2} \qquad (4)$$

Here the Hertz theory correspond to n=1. For higher values of n the pressure has its maximum values away from the centre of the contact. Persson assumed that the contact surface was cylindrical and formulated the integro-differential equation and determined the analytical contact pressure distribution for the plane stress case. The contact stress distribution for the plane stress condition is given by Persson by the expression,

$$p(\varphi) = \frac{2F(c^2-q^2)^{1/2}}{\pi R(1+q^2)(c^2+1)^{1/2}} + \frac{F(c^2+1)^{-1}}{2\pi R c^2} \times \ln\left[\frac{(c^2+1)^{1/2} + (c^2-q^2)^{1/2}}{(c^2+1)^{1/2} - (c^2-q^2)^{1/2}}\right] \qquad (5)$$

The expression for the normalized maximum pressure at the centre of contact is calculated from the eq-5. When $\phi = 0$,

$$\frac{Rp_{max}}{F} = \frac{2c}{\pi(c^2+1)^{1/2}} + \frac{\ln[(c^2+1)^{1/2}+c]}{\pi c^2(c^2+1)} \qquad (6)$$

An expression relating the contact angle α with the load F, radial difference CL, Young's modulus E is

$$\frac{E \times \overline{CL}}{F} = \frac{2(1-c^2)}{\pi c^2} + \frac{I}{\pi^2 c^2 (1+c^2)} \qquad (7)$$

Where,

$$I = \int_{-c}^{c} \frac{\ln\left[\frac{(c^2+1)^{1/2} + (c^2-q^2)^{1/2}}{(c^2+1)^{1/2} - (c^2-q^2)^{1/2}}\right]}{1+t^2}\,dt$$

The eq-7 is solved by Complex integration method and the resulting equation is given by

$$\left(\frac{\pi E \times \overline{CL}}{2F} + 1.25\right)x^2 + \left(\frac{\pi E \times \overline{CL}}{2F} - 0.5\right)x - \left(\frac{\ln 2}{\pi} + 1 - \frac{1}{2\pi}\right) = 0 \qquad (8)$$

Where $c^2 = x$, $c = \tan\left(\frac{\alpha}{2}\right)$, $q = \tan\left(\frac{\varphi}{2}\right)$, and

F = Applied load,

ϕ = The angular position,

2α = The angle of contact and

R = contacting radius.

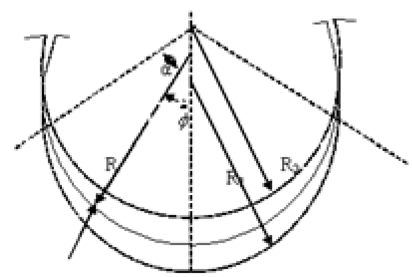

Figure 1. Cylindrical Conformal Contact

III. FINITE ELEMENT FORMULATION

Considering the symmetry of the problem the quarter of a cylinder inside a semi-infinite cylindrical cavity is taken and the non-contacting half has been neglected as shown in figure-2. Where R_1 = 10 mm, CL = 1, 0.5, 0.25 mm, X_1 = 200 mm, Y_1 = 200 mm. The dimension of the cavity is taken as large to satisfy the half space condition.

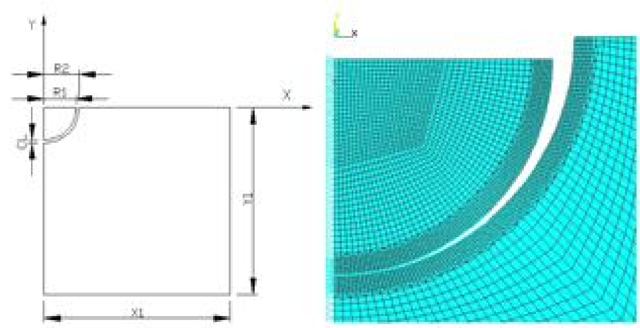

Figure 2. Geometry of the Model and mesh pattern

The bottom edge of the cavity is fixed in all direction and the left side edge is given symmetrical boundary condition. The plane stress analysis is done using Finite Element Analysis Software. The mesh near the contact area is refined sufficiently and the aspect ratio is maintained nearly equal to ideal. The analysis is carried out considering same material for the cylinder and cavity by taking the young's modulus of 2.1×10^5 N/mm^2 and Poisson's ratio of 0.3, and different material for the cylinder and cavity.The modulus index is defined for the different material for the cylinder and the cavity as the

DOI: 01.IJPIE.02.01.508

ratio of the Young's modulus of the cylinder and that of the cavity and is denoted by 'n'.

$$E_1 \neq E_2, \quad n = \frac{E_1}{E_2} \qquad (9)$$

In modeling the contact problems in the static analysis rigid body motion is the common problem. So to remove this problem modeling can be done in such a way that the contact bodies are just touching at a single point. The numbers of nodes near the contact area that may probably be come in contact is identified on two bodies and has been created the contact pair using those nodes. Both the bodies are considered as flexible one. The two node two dimensional surface-to-surface contact element is used with target element to create contact pairs. These contact elements are created on the exiting structural elements and the two dimensional surface-to-surface contact definitions are used for the analysis.

IV. RESULTS AND DISCUSSION

A. Same Young's Modulus for Two Contacting Bodies

The Young's modulus of the two contacting bodies is taken same. The results are plotted for three different clearances varied as 0.25mm, 0.5mm and 1mm at constant load. The Contact Pressure along contact surface for the various loads is as shown in fig3.

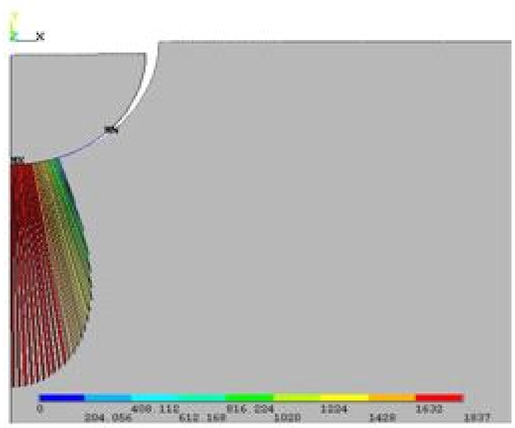

Figure 3. The Contact Pressure along contact surface for the load 10KN

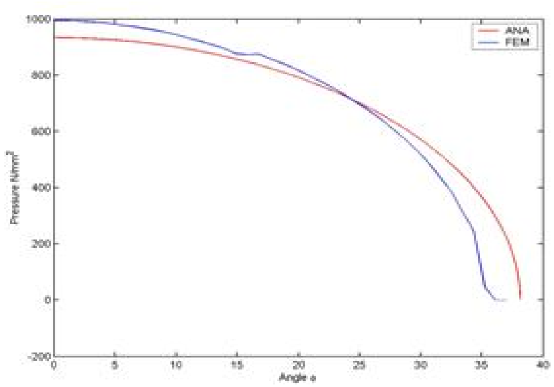

Figure 4. Variation of Contact pressure with the contact angle for the load F=10000N and 0.25mm Clearance

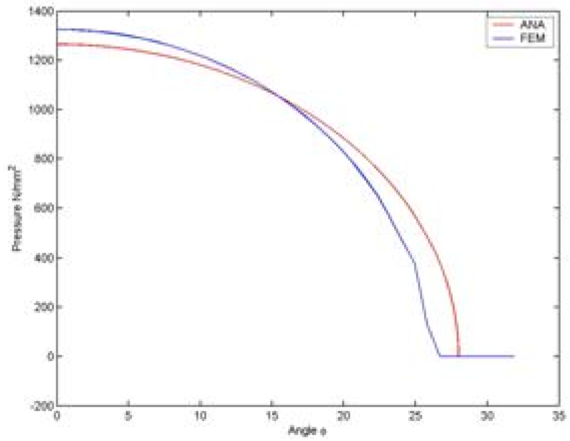

Figure 5. Variation of Contact pressure with the contact angle for the load F=10000N and 0.5mm Clearance.

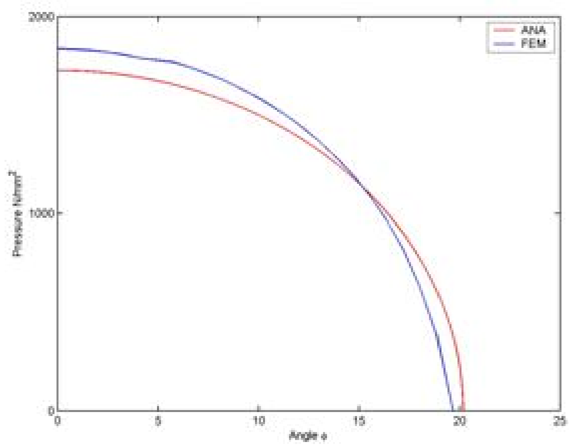

Figure 6. Variation of Contact pressure with the contact angle for the load F=10000N and 1mm Clearance.

The contact pressure increases with the increase in clearance between two contacting bodies and hence decreasing the contact angle. More the surface in contact more the stress developed due to dynamic friction as shown in fig 4 to 6.

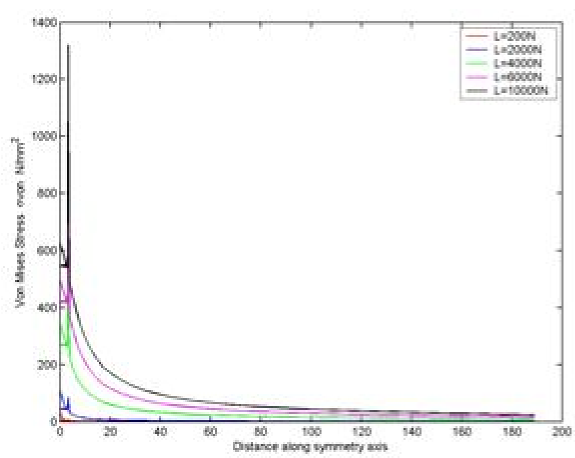

Figure 7. Von-Mises Stress along the Symmetry Axis of Cavity for Different Loads and 0.25mm Clearance.

DOI: 01.IJPIE.02.01.508

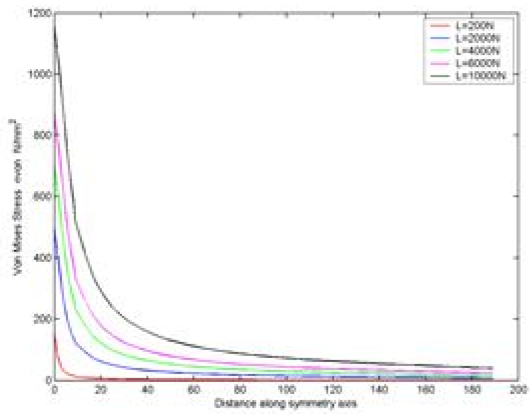

Figure 8. Von-Mises Stress along the Symmetry Axis of Cavity for Different Loads and 0.5mm Clearance

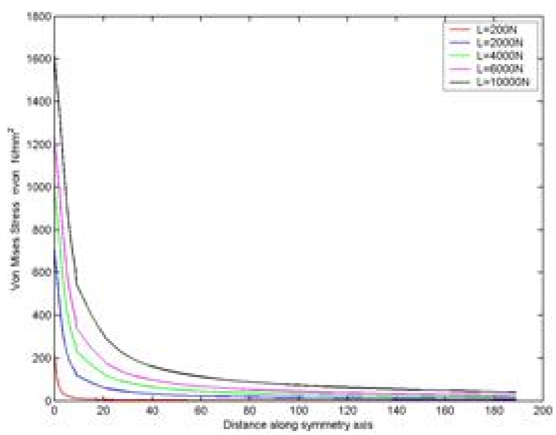

Figure 9. Von-Mises Stress along the Symmetry Axis of Cavity for Different Loads and 1mm Clearance

The stress varies with the application of load and maximum stress is developed at the center of contact region, decreases along symmetric axis as the contact angle increases as shown in Fig. 7 to 9. Lesser the clearance more the area of contact, so that more the contact angle as the load increases and visa-versa. As the load increases the cylinder undergoes deformation so that the contact angle increases is shown in fig. 10. These plots indicate that as the load increases, contact pressure also increases for the contact angle. The contact pressure decreases as the contact angle increases. This result agrees with 5% error of theory. The Von-Mises stress gives the best failure criteria for the steel materials.It is

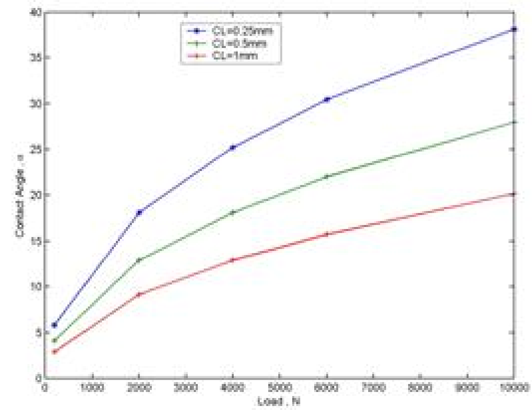

Figure 10. Variation of Contact Angle with Load for Different Clearances

understood that contact pressure increases with decrease in contact angle and as the load increases the Von-Mises stress also increases.The contact angle is half the total contact angle and is determined theoretically by the equation-8. The variation of the contact angle with the load for various values of clearances. This shows that as the load increases, the contact angle increases for the decrease of clearance.

B. Different Young's Modulus for Two Contacting Bodies

Considering different Young's modulus for cylinder and cavity (i.e. modulus index), the effect of modulus value on the contact pressure is determined. When the young's modulus of the cylinder is higher than that of the cavity (i.e. $n>1$) the contact angle increases and the contact stress decreases as shown in fig 11. The same trend is observed for the increase in load.

It is noted that as the load increases the contact angle increases and contact stress decreases for the case of same modulus value by taking the clearance is constant. If one decreases the clearance the contact angle increases for the constant load case. For a smaller contact angle and contact area the maximum pressure is greater for a higher load. All the graphs are plotted with well versed computing software MATLAB.These results are compromising well with the closed form solution obtained. Results well agree with 5% error with the theoretical values. So it can be applied to any problem of conformal contact in nature considering the material properties of contacting bodies, clearance between them and the nature of load acting on it.

DOI: 01.IJPIE.02.01.508

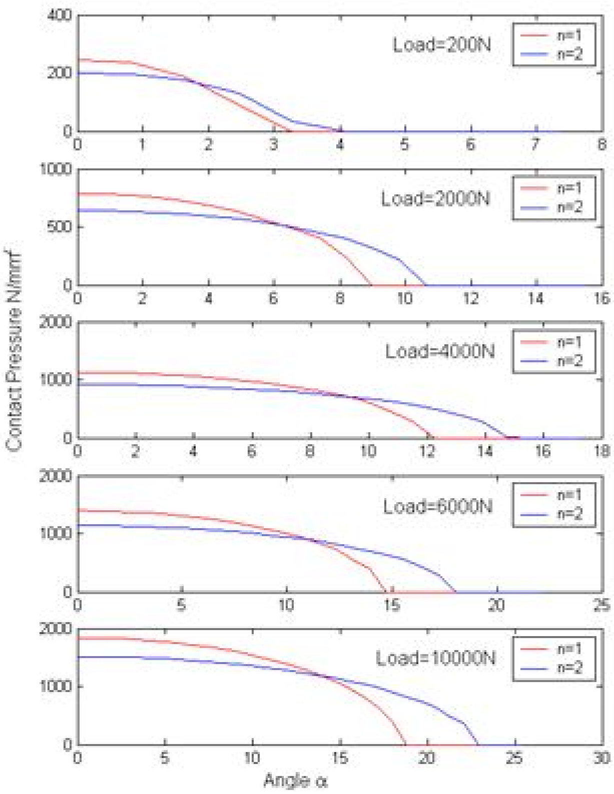

Figure 11. Variation of Contact Pressure and Contact angle for different values of Young's Modulus

CONCLUSION

It is understood that lesser the clearance between two contacting bodies, lesser the contact angle for the constant load of application. As the load increases, contact pressure also increases for the constant contact angle. The Von-Mises stress gives the best failure criteria for the steel materials asthe load increases the Von-Mises stress also increases. As the load increases the contact angle increases and contact stress also increases for the case of same modulus value by taking the clearance is constant. If one decreases the clearance the contact angle increases for the constant load case. For a smaller contact angle and contact area the maximum pressure is greater for a higher load. So it is better to have a higher the clearance is required to decrease the contact pressure and the contact angle is lesser for the increase of the modulus ratio.

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DOI: 01.IJPIE.02.01.508